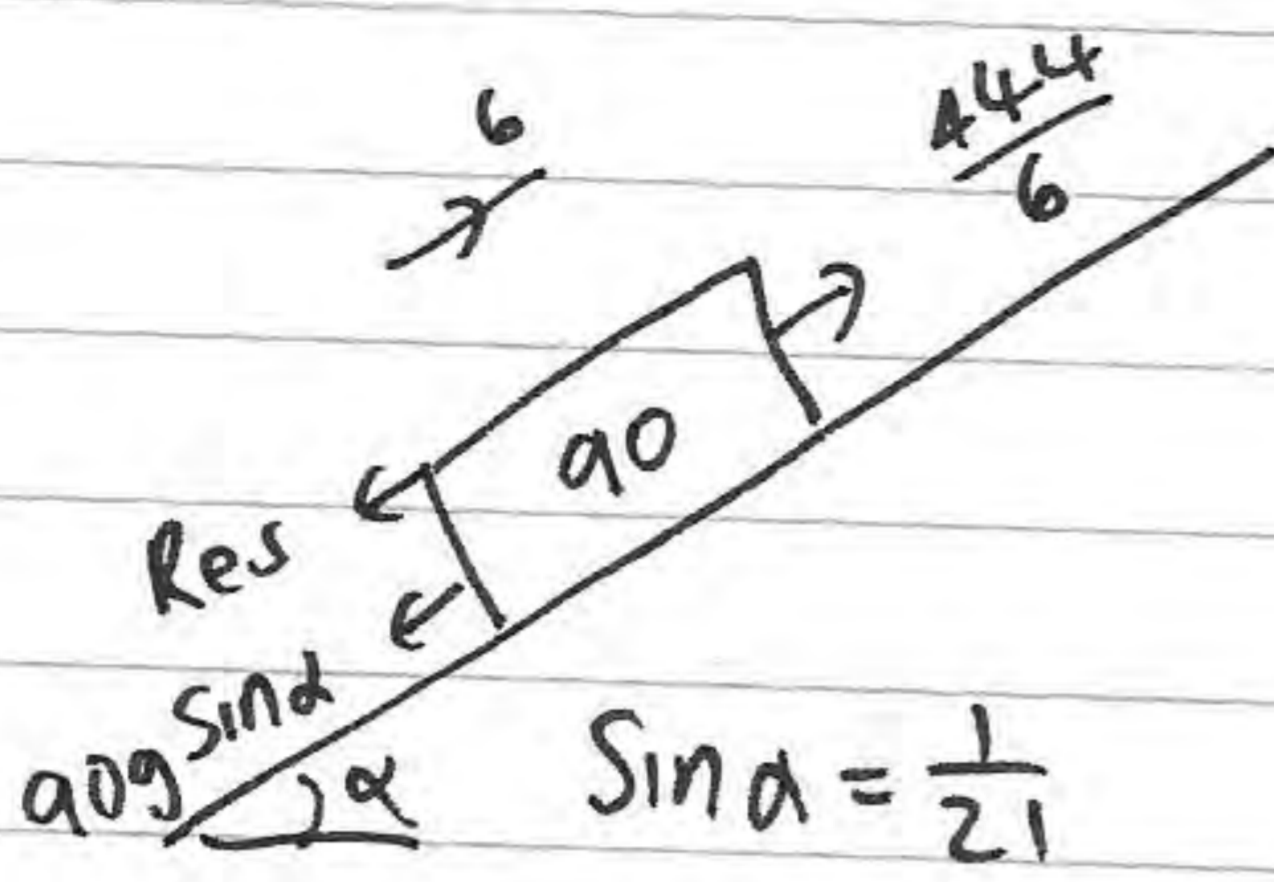
M2 JUNE 07

1)



$\sin\alpha = \frac{1}{21}$

$Rf = 0 \quad \frac{444}{6} = 90g \times \frac{1}{21} + Res$

$\Rightarrow Res = \underline{32N}$

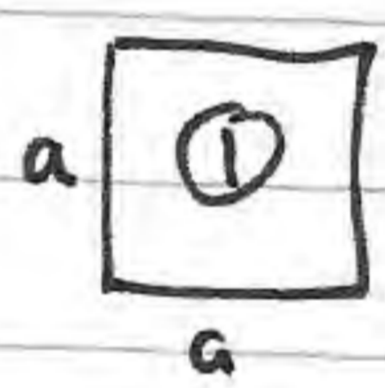
2) $V = 3t^2 i + (1-4t)j$

$a = \frac{dV}{dt} = 6t\,i - 4j$

b) $t = 2 \rightarrow a = 12i - 4j \Rightarrow f = 0.5(12i - 4j) = 6i - 2j$

$|F| = \sqrt{6^2 + 2^2} = \sqrt{40} = \underline{2\sqrt{10}\,N}$

3)



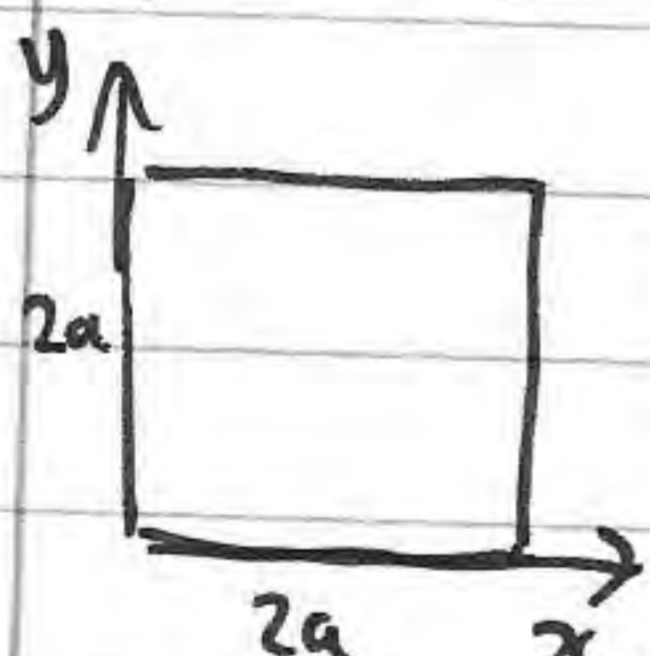
$M = a^2 k$

$g_1\left(\frac{3}{2}a, \frac{3}{2}a\right)$

$M = 3a^2 k$

$g_2(\bar{x}, \bar{y})$

$\bar{x} = \bar{y}$



$M = 4a^2 k$

$G(a, a)$

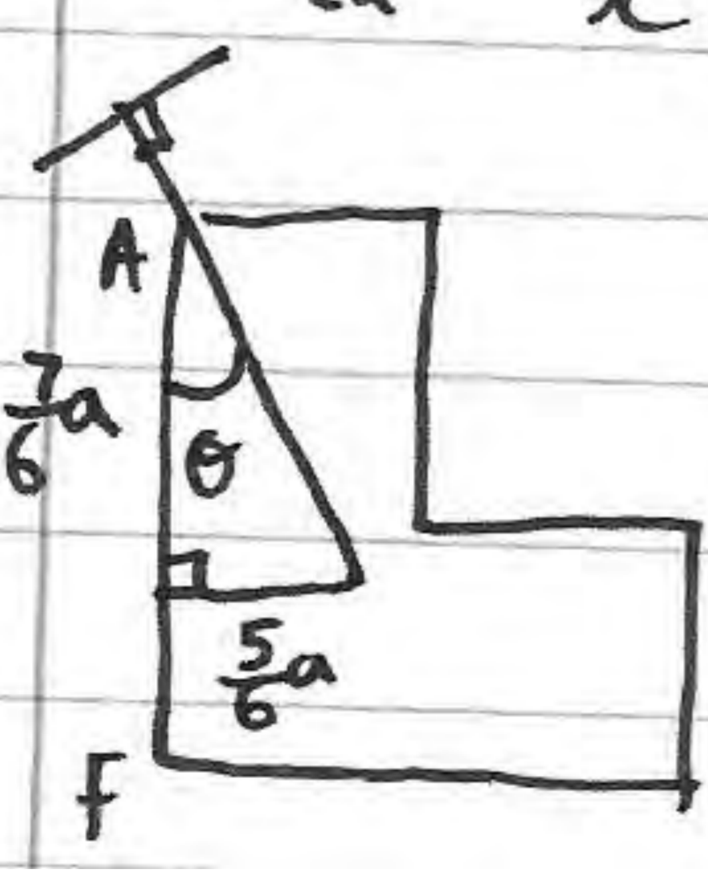
$a^2 \cancel{kg} \times \frac{3}{2}a + 3a^2 \cancel{kg}\,\bar{x} = 4a^2 \cancel{kg} \times a$

$\Rightarrow \frac{3}{2}a^3 + 3a^2\bar{x} = 4a^3$

$3a^{\cancel{2}}\bar{x} = \frac{5}{2}a^{\cancel{3}}$

$\bar{x} = \frac{5}{6}a \qquad \bar{y} = \frac{5}{6}a$

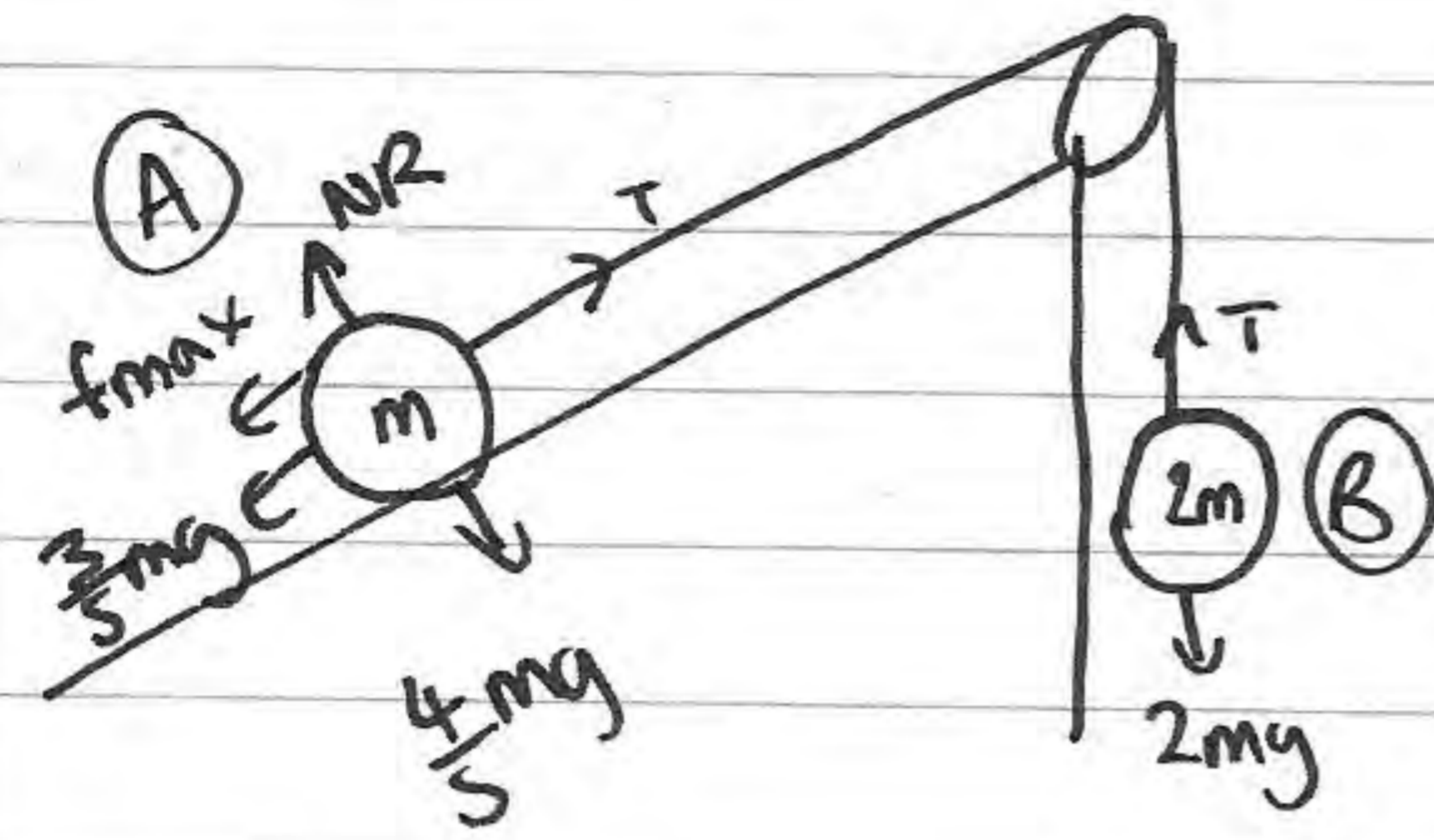
$g_2\left(\frac{5}{6}a, \frac{5}{6}a\right)$

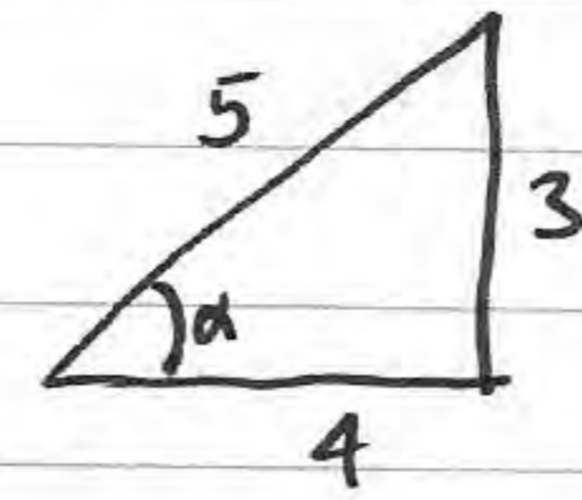


$\theta = \tan^{-1}\left(\frac{\frac{5}{\cancel{6}}\cancel{a}}{\frac{7}{\cancel{6}}\cancel{a}}\right) = \tan^{-1}\left(\frac{5}{7}\right)$

$\theta = \underline{35.5°}$

4)





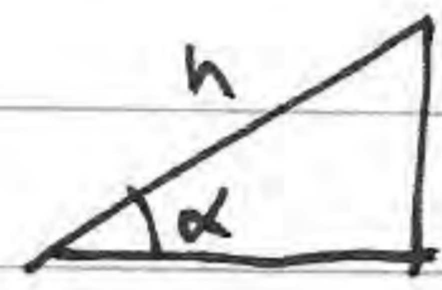
$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\mu = \frac{5}{8}$

a)



$h\sin\alpha = \frac{3}{5}h$

PE gained in A $= mg\left(\frac{3}{5}h\right) = \frac{3}{5}mgh$

PE lost in B $= (2m)gh = 2mgh$

$\therefore$ PE lost in System $= \underline{\frac{7}{5}mgh}$

PE lost = KE GAIN + Wd against friction

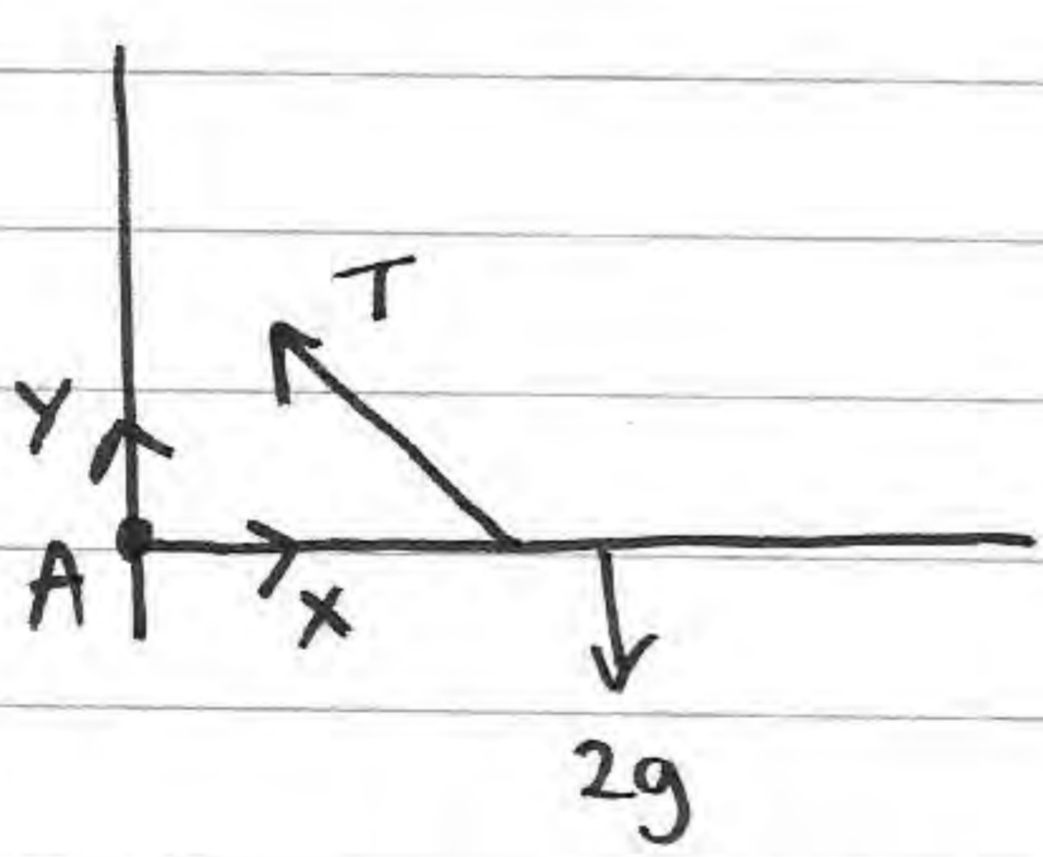
$\frac{7}{5}mgh = \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 + f_{max} \times h$ $f_{max} = \frac{5}{8} \times \frac{4}{5}mg$

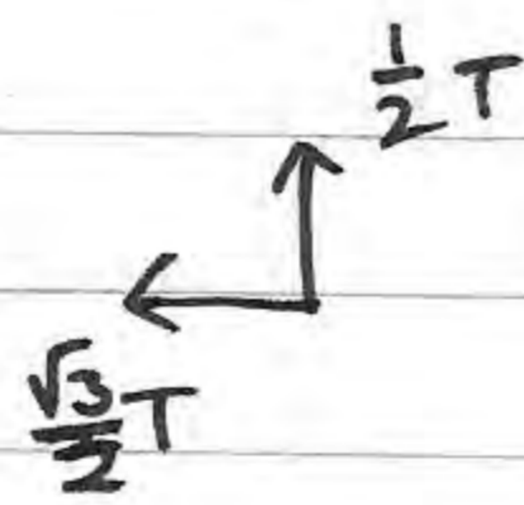
$\Rightarrow \frac{7}{5}\cancel{m}gh = \frac{3}{2}\cancel{m}v^2 + \frac{1}{2}\cancel{m}gh$

$\times 10 \Rightarrow 14gh = 15v^2 + 5gh \Rightarrow 15v^2 = 9gh \Rightarrow \underline{v^2 = \frac{9}{15}gh}$

$\therefore \underline{v^2 = \frac{3}{5}gh}$

5)





$R\overrightarrow{f} = 0 \quad X = \frac{\sqrt{3}}{2}T$

$Rf\uparrow = 0 \quad \frac{1}{2}T + Y = 2g$

$A\circlearrowleft \quad 2g \times \left(\frac{1}{2}AB\right) = \frac{1}{2}T \times 0.14$ $T = 63$

$\Rightarrow AB = \frac{4.41}{g} = \underline{0.45m\ (2sf)}$

$X = \frac{63\sqrt{3}}{2} \quad Y = 2g - 31.5$

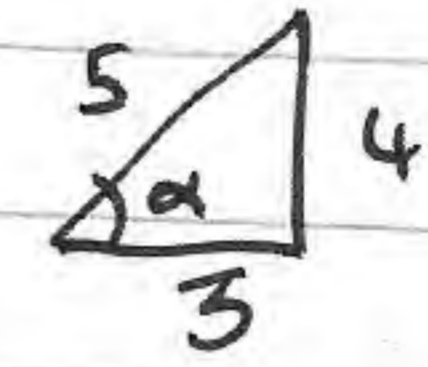
$Res = \sqrt{\left(\frac{63\sqrt{3}}{2}\right)^2 + (2g - 31.5)^2} = \underline{55.8N}\ (3sf)$

6) $V\uparrow$ $\uparrow u = 35\sin\alpha = 28$

$\uparrow v = 0$

$\uparrow a = -9.8$

$\tan\alpha = \frac{4}{3}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$0 = 28^2 - 19.6s \Rightarrow s = \underline{40m}$ above A

b) $\overrightarrow{H}$ $\vec{u} = 35\cos\alpha = 21$

$x = 168$

$168 = 21t \Rightarrow t = 8$ sec

$V\uparrow$

$u = 28$

$a = -9.8$

$t = 8$

$s = 28 \times 8 - 4.9 \times 8^2 = -89.6m$

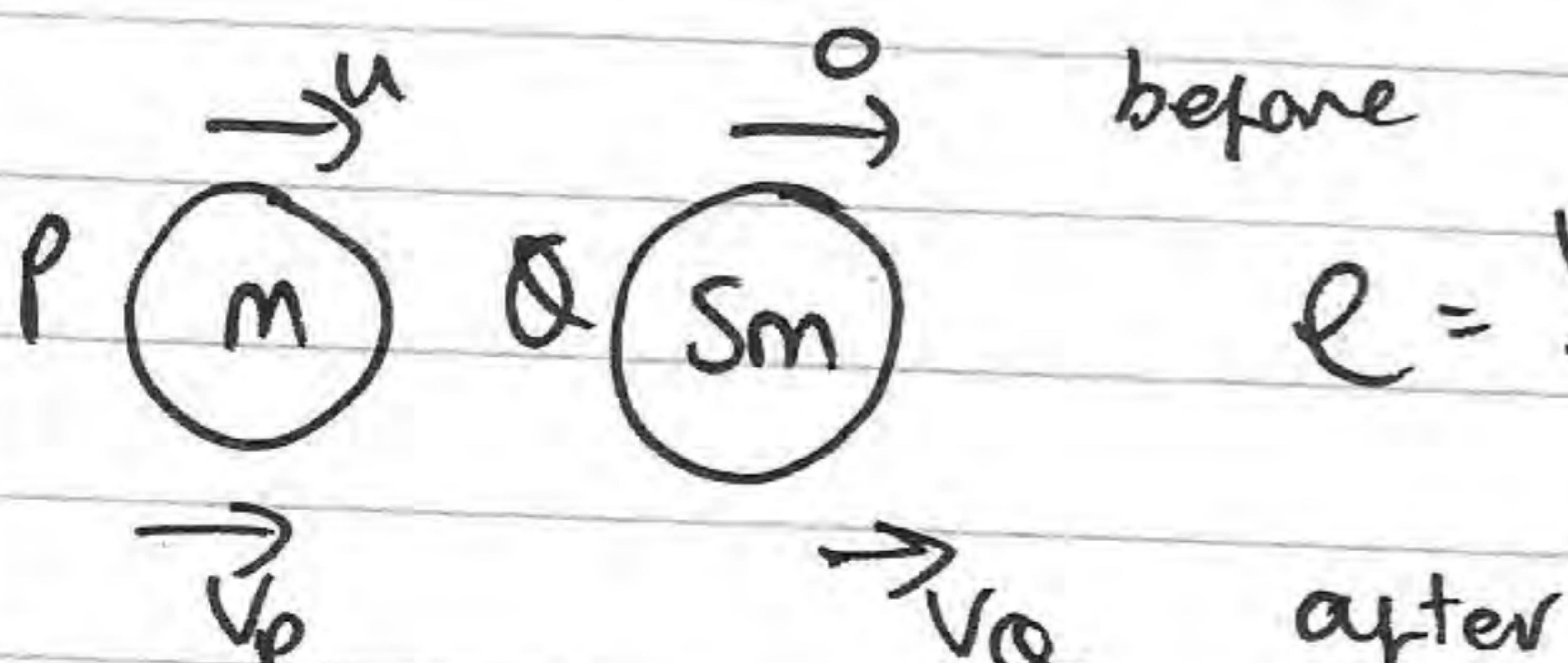
A is $\underline{89.6m}$ above the ground

c) gain in KE = loss in PE

$\frac{1}{2}\cancel{m}(35)^2 + \cancel{m}g(89.6) = \frac{1}{2}\cancel{m}v^2 \Rightarrow v^2 = \frac{74529}{25}$

$\Rightarrow v = \underline{54.6ms^{-1}}$ (3sf

7)



$e = \frac{V_Q - V_P}{u} \Rightarrow V_Q - V_P = eu$

$V_Q = V_P + eu$

$$CLM \Rightarrow mu = mV_P + 5mV_Q \qquad V_Q = V_P + eu$$

$$\Rightarrow u = V_P + 5V_P + 5eu$$

$$\Rightarrow 6V_P = u - 5eu$$

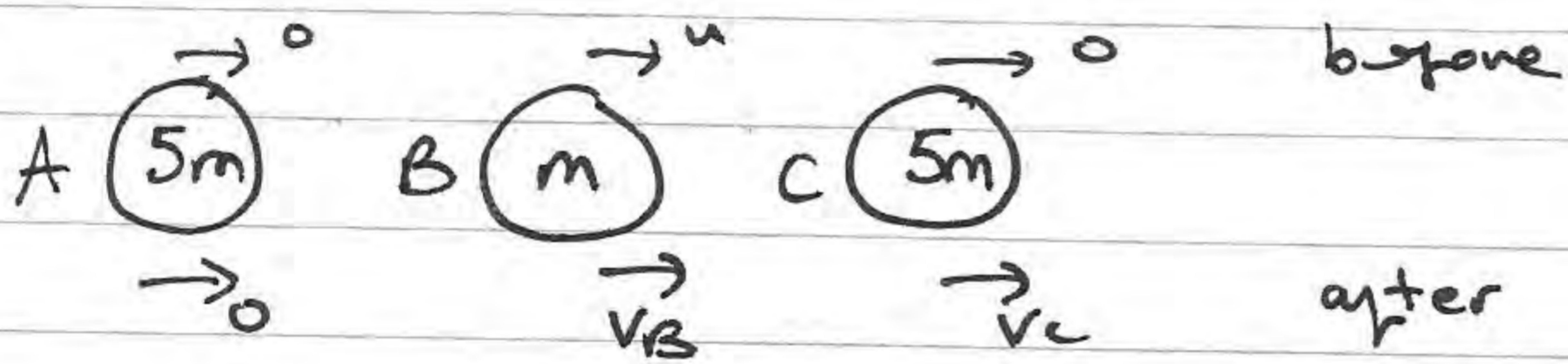
$$\Rightarrow V_P = \tfrac{1}{6}u(1-5e) \Rightarrow V_P = -\tfrac{1}{6}u(5e-1)$$

$$\therefore \text{Speed} = \tfrac{1}{6}u(5e-1)$$

ii) $V_Q = V_P + eu = -\frac{1}{6}u(5e-1) + eu = -\frac{5}{6}eu + eu + \frac{1}{6}u$

$$V_Q = \tfrac{1}{6}eu + \tfrac{1}{6}u \Rightarrow V_Q = \tfrac{1}{6}u(e+1) \qquad \lambda = \frac{e+1}{6}$$

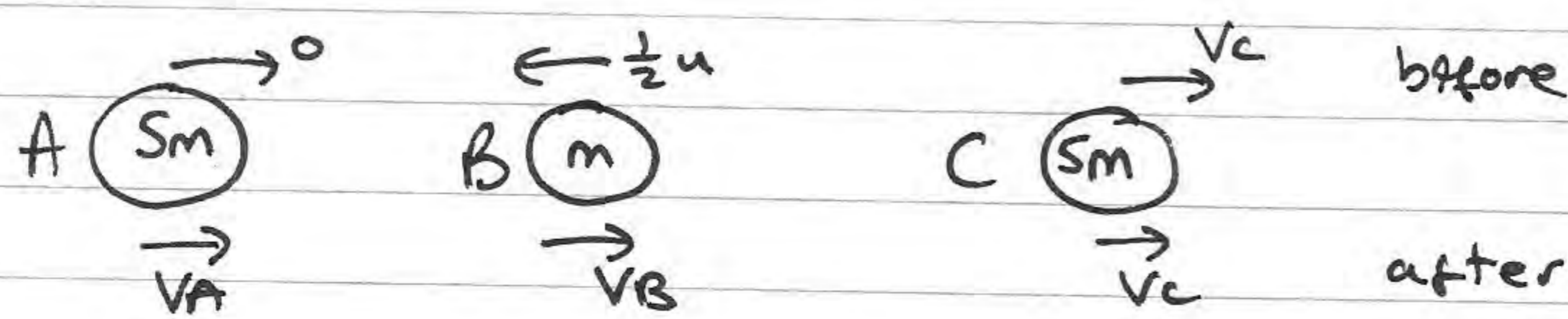
b)



from (a) $V_B = -\frac{1}{6}u(5e-1) \quad e = \frac{4}{5} \quad V_B = -\frac{3}{6}u = -\frac{1}{2}u$

$V_B = \leftarrow \frac{1}{2}u$ so it will collide with A

(c)



$$e_{AB} = \frac{V_B - V_A}{\frac{1}{2}u} = \frac{4}{5} \Rightarrow 5V_B - 5V_A = 2u$$

$$CLM_{AB} \quad -\frac{1}{2}mu = 5mV_A + \cancel{5}mV_B$$

$$\Rightarrow -\frac{1}{2}mu =$$

$$e_{AB} = \frac{V_B - V_A}{\frac{1}{2}u} = \frac{4}{5} \Rightarrow 5V_B - 5V_A = 2u$$

$$\Rightarrow 5V_A = 5V_B - 2u$$

$$CLM_{AB} \Rightarrow -\frac{1}{2}\cancel{m}u = 5\cancel{m}V_A + \cancel{m}V_B$$

$$\Rightarrow -\frac{1}{2}u = 5V_B - 2u + V_B$$

$$\Rightarrow \frac{3}{2}u = 6V_B \qquad \Rightarrow V_B = \frac{3}{12}u = \frac{1}{4}u$$

from(a) $V_C = \frac{1}{6}u(e+1)$ $\quad e = \frac{4}{5}$ $\quad V_C = \frac{1}{6}u\left(\frac{9}{5}\right)$

$$\Rightarrow V_C = \frac{3}{10}u$$

$$V_B = \frac{1}{4}u \qquad V_C = \frac{3}{10}u$$

Since $\frac{1}{4}u < \frac{3}{10}u$ B and C will <u>not</u> collide again.

8) greatest speed when $acc = \frac{dv}{dt} = 0$

$\frac{dv}{dt} = 8 - 3t = 0 \qquad 3t = 8 \qquad t = \frac{8}{3}$

$V = 8\left(\frac{8}{3}\right) - \frac{3}{2}\left(\frac{8}{3}\right)^2 = \underline{\frac{32}{3}\ ms^{-1}}$

b) $S = \int v\,dt = 4t^2 - \frac{1}{2}t^3 + C \qquad t=0, S=0 \Rightarrow C=0$

$S = 4t^2 - \frac{1}{2}t^3, \quad t=4 \qquad S = 32m$

c) at rest when $v = 0 \Rightarrow t = 8$

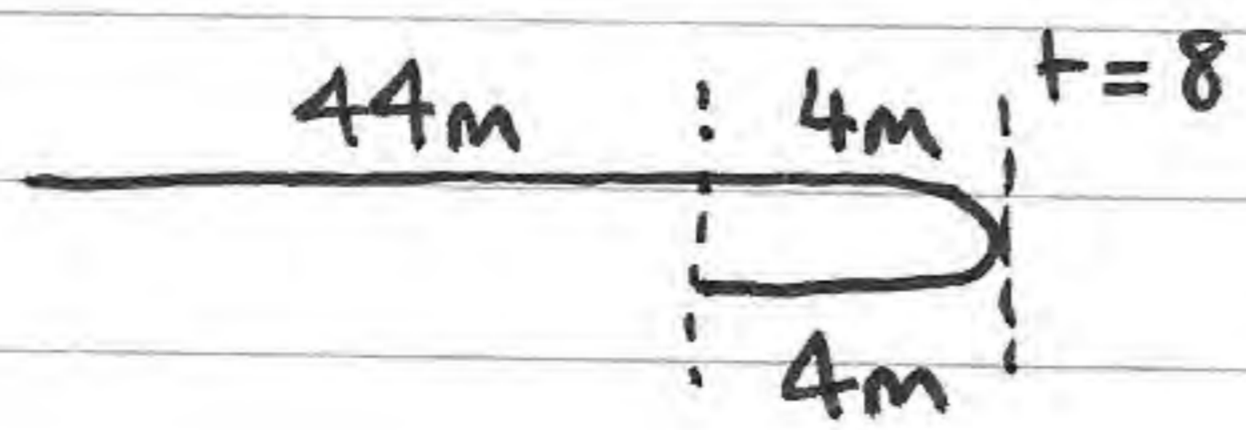
d) $S = \int v\,dt = 16t - t^2 + C \qquad t=4, S=32$

$\Rightarrow 32 = 64 - 16 + C \Rightarrow C = -16$

$S = 16t - t^2 - 16 \qquad$ when $t = 10 \qquad S = 44m$

<u>displacement</u> from O $= 44m$ but the particle travels towards the origin after 8 sec.

$t = 8 \qquad S = 16(8) - 8^2 - 16 = 48m$



$\therefore$ total <u>distance</u> travelled $= 44 + 4 + 4 = \underline{52m}$